

Time-like Extra Dimensions without Tachyons or Ghosts

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Abstract

Some time ago Dvali, Gabadadze and Senjanović [1] discussed brane world scenarios with time-like extra dimensions. In this paper we construct a solitonic 3-brane solution in the 5-dimensional Einstein-Hilbert-Gauss-Bonnet theory with the space-time signature $(-, +, +, +, -)$. The direction transverse to the brane is the second time-like direction. The solitonic brane is δ -function like, and has the property that gravity is completely localized on the brane. That is, there are no propagating degrees of freedom in the bulk, while on the brane we have purely 4-dimensional Einstein gravity. In particular, there are no propagating tachyonic or negative norm states even though the extra dimension is time-like.

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I. INTRODUCTION

In the Brane World scenario the Standard Model gauge and matter fields are assumed to be localized on branes (or an intersection thereof), while gravity lives in a larger dimensional bulk of space-time [2–18]. Usually it is assumed that the extra dimensions transverse to the branes are space-like. This is because otherwise we generically expect difficulties with propagating tachyonic and/or negative norm states whose appearance is due to the presence of more than one time-like directions. Various issues arising in brane world scenarios with time-like extra dimensions were discussed in [1] (for subsequent developments, see [19]).

In this paper we would like to ask whether the aforementioned difficulties can be avoided in brane world scenarios with time-like extra dimensions. As we will argue in the following, the answer to this question appears to be positive. Thus, recently in [20] we constructed a (flat) *solitonic* codimension-one brane world solution (with a space-like extra dimension), where gravity is completely localized on the brane. That is, the graviton propagator in the bulk *vanishes*, while it is non-trivial on the brane. In this paper we point out that such a solution also exists in the case where the extra dimension is *time-like*. In this solution we also have completely localized gravity, and we have no propagating tachyonic or negative norm states as there are no propagating degrees of freedom in the bulk. Moreover, just as in the solution of [20], albeit the classical background is 5-dimensional, the quantum theory (at least perturbatively) is actually 4-dimensional. In particular, there are no loop corrections in the bulk.

The setup within which we construct this solitonic brane world solution is the 5-dimensional Einstein-Hilbert theory with a (positive) cosmological term augmented with a Gauss-Bonnet coupling (the signature of the 5-dimensional space is $(-, +, +, +, -)$). The solitonic brane world solution arises in this theory for a special value of the Gauss-Bonnet coupling. The fact that there are no propagating degrees of freedom in the bulk is then due to a perfect cancellation between the corresponding contributions coming from the Einstein-Hilbert and Gauss-Bonnet terms. Since the bulk theory does not receive loop corrections, the classical choice of parameters such as the special value of the Gauss-Bonnet coupling (or the Gauss-Bonnet combination itself) does *not* require order-by-order fine-tuning. Also, we can embed this solution in the (minimally) supersymmetric setup, where we still have no propagating degrees of freedom in the bulk, while on the brane we have completely localized *supergravity*. Here the solitonic brane is a BPS solution (with vanishing brane cosmological constant), which preserves 1/2 of the original supersymmetries.

II. THE SETUP

In this section we discuss the setup within which we will discuss the aforementioned solitonic brane world solution. The action for this model is given by (for calculational convenience we will keep the number of space-time dimensions D unspecified):

$$S = M_P^{D-2} \int d^D x \sqrt{-G} (R - \Lambda + \lambda [R^2 - 4R_{MN}^2 + R_{MNST}^2]) , \quad (1)$$

where M_P is the D -dimensional (reduced) Planck scale, and the Gauss-Bonnet coupling λ has dimension (length)². Finally, the bulk vacuum energy density Λ is a constant. The D -dimensional space-time has signature $(-, +, \dots, +, -)$.

In the following we will be interested in solutions to the equations of motion following from the action (1) with the warped [21] metric of the form

$$ds_D^2 = \exp(2A)\eta_{MN}dx^Mdx^N , \quad (2)$$

where $\eta_{MN} \equiv \text{diag}(-1, +1, \dots, +1, -1)$, and the warp factor A , which is a function of $z \equiv x^D$, is independent of the other $(D-1)$ coordinates x^μ . With this Ansatz, we have the following equations of motion for A (prime denotes derivative w.r.t. z):

$$(D-1)(D-2)(A')^2 [1 + \kappa] - 2\Lambda \exp(2A) = 0 , \quad (3)$$

$$(D-2) [A'' - (A')^2] \kappa = 0 , \quad (4)$$

where

$$\kappa \equiv 1 + 2(D-3)(D-4)\lambda(A')^2 \exp(-2A) . \quad (5)$$

For generic values of Λ and λ such that the above system of equations has a solution, the corresponding background is expected to suffer from propagating negative norm states as we have two time-like directions.

There, however, also exists a solution which is free of propagating negative norm states. Thus, consider the case where

$$\Lambda = -\frac{(D-1)(D-2)}{(D-3)(D-4)} \frac{1}{4\lambda} \quad (6)$$

with $\lambda < 0$ and $\Lambda > 0$. Then we have the following solution (we have chosen the integration constant such that $A(0) = 0$):

$$A(z) = -\ln \left[\frac{|z|}{\Delta} + 1 \right] , \quad (7)$$

where Δ is given by

$$\Delta^2 = -2(D-3)(D-4)\lambda . \quad (8)$$

Note that Δ can be positive or negative. In the former case the volume of the z direction is finite: $v = 2\Delta/(D-1)$. On the other hand, in the latter case it is infinite. As we will see in the following, the negative Δ case corresponds to a non-unitary theory.

Note that A' is discontinuous at $z = 0$, and A'' has a δ -function-like behavior at $z = 0$. Note, however, that (4) is still satisfied as in this solution

$$\kappa = 0 . \quad (9)$$

Thus, this solution describes a codimension one *soliton* with a time-like transverse dimension. The tension of this soliton, which is given by

$$f = -\frac{4(D-2)}{\Delta} M_P^{D-2} , \quad (10)$$

is negative for $\Delta > 0$, and it is positive for $\Delta < 0$. The aforementioned non-unitarity in the latter case is, in fact, attributed to the positivity of the brane tension (but has nothing to do with the fact that we have two time-like dimensions). Note that this is opposite to the case with a space-like extra dimension [20]. Here and in the following we refer to the $z = 0$ hypersurface, call it Σ , as the brane. Note that the background metric on the brane is the flat $(D-1)$ -dimensional Minkowski metric $\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1)$.

III. GRAVITY IN THE SOLITONIC BRANE WORLD

In this section we would like to study gravity in the solitonic brane world solution discussed in the previous section along the lines of [20]. Let us study small fluctuations around the solution:

$$G_{MN} = \exp(2A) \left[\eta_{MN} + \tilde{h}_{MN} \right] , \quad (11)$$

where for convenience reasons we have chosen to work with \tilde{h}_{MN} instead of metric fluctuations $h_{MN} = \exp(2A)\tilde{h}_{MN}$.

Let us assume that we have matter localized on the brane, and let the corresponding conserved energy-momentum tensor be $T_{\mu\nu}$:

$$\partial^\mu T_{\mu\nu} = 0 . \quad (12)$$

The graviton field $\tilde{h}_{\mu\nu}$ couples to $T_{\mu\nu}$ via the following term in the action (note that $\tilde{h}_{\mu\nu} = h_{\mu\nu}$ at $z = 0$ as we have set $A(0) = 0$):

$$S_{\text{int}} = \frac{1}{2} \int_{\Sigma} d^{D-1}x \ T_{\mu\nu} \tilde{h}^{\mu\nu} . \quad (13)$$

In the following we will use the following notations for the component fields:

$$H_{\mu\nu} \equiv \tilde{h}_{\mu\nu} , \quad A_\mu \equiv \tilde{h}_{\mu D} , \quad \rho \equiv \tilde{h}_{DD} . \quad (14)$$

The linearized equations of motion for the component fields $H_{\mu\nu}$, A_μ and ρ read:

$$\begin{aligned} & \kappa \left(\Omega_{\mu\nu} - \Sigma'_{\mu\nu} - (D-2)A'\Sigma_{\mu\nu} - \partial_\mu \partial_\nu \rho + \eta_{\mu\nu} \partial_\sigma \partial^\sigma \rho + \right. \\ & \eta_{\mu\nu} \left[(D-2)A'\rho' + (D-1)(D-2)(A')^2 \rho \right] \Big) + \\ & 4(D-4)\lambda \left[A'' - (A')^2 \right] e^{-2A} \left(\Omega_{\mu\nu} - (D-3)A'\Sigma_{\mu\nu} \right) + \\ & 2(D-2) [2\kappa - 1] \left[A'' - (A')^2 \right] \eta_{\mu\nu} \rho = \\ & -M_P^{2-D} T_{\mu\nu} \delta(z) , \end{aligned} \quad (15)$$

$$\kappa \left(Q'_\nu - \partial^\mu F_{\mu\nu} - (D-2)A' \partial_\nu \rho \right) = 0 , \quad (16)$$

$$\kappa \left(\partial^\nu Q_\nu + A'\Sigma - (D-1)(D-2)(A')^2 \rho \right) = 0 , \quad (17)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the $U(1)$ field strength for the graviphoton,

$$\begin{aligned}\Omega_{\mu\nu} &\equiv \partial_\sigma \partial^\sigma H_{\mu\nu} + \partial_\mu \partial_\nu H - \partial_\mu \partial^\sigma H_{\sigma\nu} - \partial_\nu \partial^\sigma H_{\sigma\mu} - \\ &\quad \eta_{\mu\nu} [\partial_\sigma \partial^\sigma H - \partial^\sigma \partial^\rho H_{\sigma\rho}] ,\end{aligned}\tag{18}$$

$$\Sigma_{\mu\nu} \equiv H'_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - \eta_{\mu\nu} [H' - 2\partial^\sigma A_\sigma] ,\tag{19}$$

$$Q_\nu \equiv \partial^\mu H_{\mu\nu} - \partial_\nu H ,\tag{20}$$

while $H \equiv H_\mu^\mu$, and $\Sigma \equiv \Sigma^\mu_\mu$.

The above equations of motion are invariant under certain gauge transformations corresponding to unbroken diffeomorphisms. In terms of the component fields $H_{\mu\nu}$, A_μ and ρ , the full D -dimensional diffeomorphisms read:

$$\delta H_{\mu\nu} = \partial_\mu \tilde{\xi}_\nu + \partial_\nu \tilde{\xi}_\mu - 2\eta_{\mu\nu} A' \omega ,\tag{21}$$

$$\delta A_\mu = \tilde{\xi}'_\mu + \partial_\mu \omega ,\tag{22}$$

$$\delta \rho = 2\omega' + 2A' \omega ,\tag{23}$$

where $\omega \equiv \tilde{\xi}_D$. It is not difficult to check that the equations of motion (15), (16) and (17) are invariant under these full D -dimensional diffeomorphisms. That is, there are no restrictions on ω or $\tilde{\xi}_\mu$ or derivatives thereof including at $z = 0$. In particular, this is the case for the solitonic brane world solution despite its δ -function-like structure. The reason for this is that this solution being a soliton does not break the full D -dimensional diffeomorphisms explicitly but only *spontaneously*.

Since we have the full D -dimensional diffeomorphisms, we can always gauge A_μ and ρ away. In fact, in the following we will see that for the solitonic brane world background this can indeed be done without introducing any inconsistencies. However, before we adapt this gauge fixing, we would like to make the following important observation. Note that for the solitonic brane world solution (7) with Δ given by (8) we have (9). On the other hand, this vanishing factor κ is precisely the one that multiplies the terms in (15), (16) and (17) corresponding to the propagation of the fields $H_{\mu\nu}$, A_μ and ρ in the bulk. That is, in the solitonic brane world solution these fields do *not* propagate in the time-like z direction at all. This is due to a cancellation between contributions of the Einstein-Hilbert and Gauss-Bonnet terms into the bulk propagator in this background. On the other hand, (some of) these fields do propagate on the brane. Indeed, in the above background we have

$$A'' - (A')^2 = -\frac{2}{\Delta} \delta(z) .\tag{24}$$

Then (15) gives the following equation of motion (note that (16) and (17) are trivially satisfied in this background):

$$\begin{aligned}\left(\Omega_{\mu\nu} - (D-3)A' \Sigma_{\mu\nu} - \frac{(D-2)(D-3)}{\Delta^2} \eta_{\mu\nu} \rho \right) \delta(z) = \\ -\widehat{M}_P^{3-D} T_{\mu\nu} \delta(z) ,\end{aligned}\tag{25}$$

where

$$\widehat{M}_P^{D-3} \equiv \frac{4\Delta}{D-3} M_P^{D-2} ,\tag{26}$$

and in the following we will identify \widehat{M}_P with the $(D - 1)$ -dimensional Planck scale.

Thus, as we see, in the negative tension solution there are no propagating tachyonic or negative norm states in the bulk or on the brane, and the theory is unitary. Moreover, since we have no propagating degrees of freedom in the bulk, we have no loop corrections in the bulk either. This implies that, albeit the classical background is D -dimensional, the quantum theory (at least perturbatively) is actually $(D - 1)$ -dimensional. In particular, the condition (6) is stable against loop corrections.

A. Completely Localized Gravity

Next, we would like to see what is the solution to the equation of motion (25). First, note that, as we have already mentioned, we can always gauge A_μ and ρ away. That is, these fields are *not* propagating degrees of freedom. Note that after this gauge fixing the residual gauge symmetry is given by the $(D - 1)$ -dimensional diffeomorphisms for which $\omega \equiv 0$, and ξ_μ are independent of z . Second, note that the second term in parentheses on the l.h.s. of (25) contains $A'\delta(z)$. This quantity, however, is vanishing as A' has a $\text{sign}(z)$ -like discontinuity at $z = 0$. We therefore obtain the following equation of motion for the $(D - 1)$ -dimensional graviton components $H_{\mu\nu}$:

$$(\Omega_{\mu\nu} + \widehat{M}_P^{3-D} T_{\mu\nu})\delta(z) = 0 . \quad (27)$$

Note that this equation is purely $(D - 1)$ -dimensional. Thus, gravity is *completely* localized on the brane, that is at the $z = 0$ hypersurface Σ . In particular, the graviton field $H_{\mu\nu}$ is non-vanishing only on the brane, while it vanishes in the bulk:

$$H_{\mu\nu}(z \neq 0) = 0 . \quad (28)$$

Note that (27) does not by itself imply (28). In particular, *a priori* $H_{\mu\nu}$ at $z \neq 0$ can be arbitrary. However, as we explained above, we have no propagating degrees of freedom in the bulk, that is, the graviton propagator in the bulk vanishes, while it is non-vanishing only on the brane. This implies that perturbations due to matter localized on the brane should *not* propagate into the bulk but only on the brane, hence (28).

On the brane (27) can be solved in a standard way. From (27) it is clear that \widehat{M}_P is the $(D - 1)$ -dimensional Planck scale for $(D - 1)$ -dimensional gravity localized on the brane. Actually, \widehat{M}_P is identified with the $(D - 1)$ -dimensional Planck scale for the positive Δ solution. As to the negative Δ solution, we have “antigravity” localized on the brane, and the corresponding theory is non-unitary due to negative norm states propagating on the brane.

Note that above our analysis was confined to the linearized theory. The above conclusions, however, are valid in the full non-linear theory. Indeed, we have no propagating degrees of freedom in the bulk, while on the brane we have only the zero mode for the $(D - 1)$ -dimensional graviton components $H_{\mu\nu}$. This then implies that in the solitonic brane world background (the gravitational part of) the brane world-volume theory is described by the $(D - 1)$ -dimensional Einstein-Hilbert action:

$$S_{\text{brane}} = \widehat{M}_P^{D-2} \int d^{D-1}x \sqrt{-\widehat{G}} \widehat{R} , \quad (29)$$

where $\hat{G}_{\mu\nu}$ is the $(D - 1)$ -dimensional metric on the brane; all the hatted quantities are $(D - 1)$ -dimensional, and are constructed from $\hat{G}_{\mu\nu}$. Note that there is no $(D - 1)$ -dimensional Gauss-Bonnet term in this action, which can be seen by examining the full non-linear equations of motion following from (1).

IV. COMMENTS

We would like to end our discussion with the following remarks. As in the case of a solitonic brane world solution of [20] with a space-like extra dimension, in the above solution with a time-like extra dimension there exist consistent curved deformations (that is, solutions with non-vanishing brane cosmological constant on the brane). However, if we embed our solution in the (minimal) supergravity framework (such an embedding exists in complete parallel with the solution discussed in [20]), then the corresponding solitonic brane world is a BPS solution which preserves $1/2$ of the original supersymmetries. The $(D - 1)$ -dimensional cosmological constant on such a BPS brane is then necessarily vanishing.

Let us point out that, just as is the case for the solitonic brane world solution of [20], the solution discussed in this paper does not suffer from the difficulties such as delocalization of gravity [22–24] or inconsistency of the coupling between brane matter and bulk gravity [23,25], which are generically expected to occur at the quantum level in warped backgrounds such as [15]¹.

In the above solitonic solution perturbatively we have no propagating degrees of freedom in the bulk. At first this might appear to imply that the extra dimension is immaterial. Note, however, that, as was pointed out in [20], non-perturbative corrections in the bulk can have non-trivial implications. In particular, semi-classically causality can be broken via creation of “baby branes” (which, nonetheless, need not violate unitarity even if we have a time-like extra dimension). An interesting phenomenological implication of this would be violation of global quantum numbers such as the baryon and lepton numbers along the lines of [26].

Finally, in the above setup the classical choice of parameters given by (6) is *necessary* to ensure unitarity as the extra dimension is time-like, so this choice is *not* a fine-tuning, but rather is required by unitarity². On the other hand, (6) is stable against loop corrections both in the setup of this paper as well as that of [20].

¹Here we should point out that, as in the model of [20], we can construct the above δ -function-like solitonic solution as a limit of a thick solitonic domain wall. As was discussed in detail in [20], in the corresponding solution only the graviton zero mode is a propagating solution.

²Here we would like to point out that, as was discussed in [27], the corresponding choice of parameters in the case of a space-like extra dimension is *not* a fine-tuning either.

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